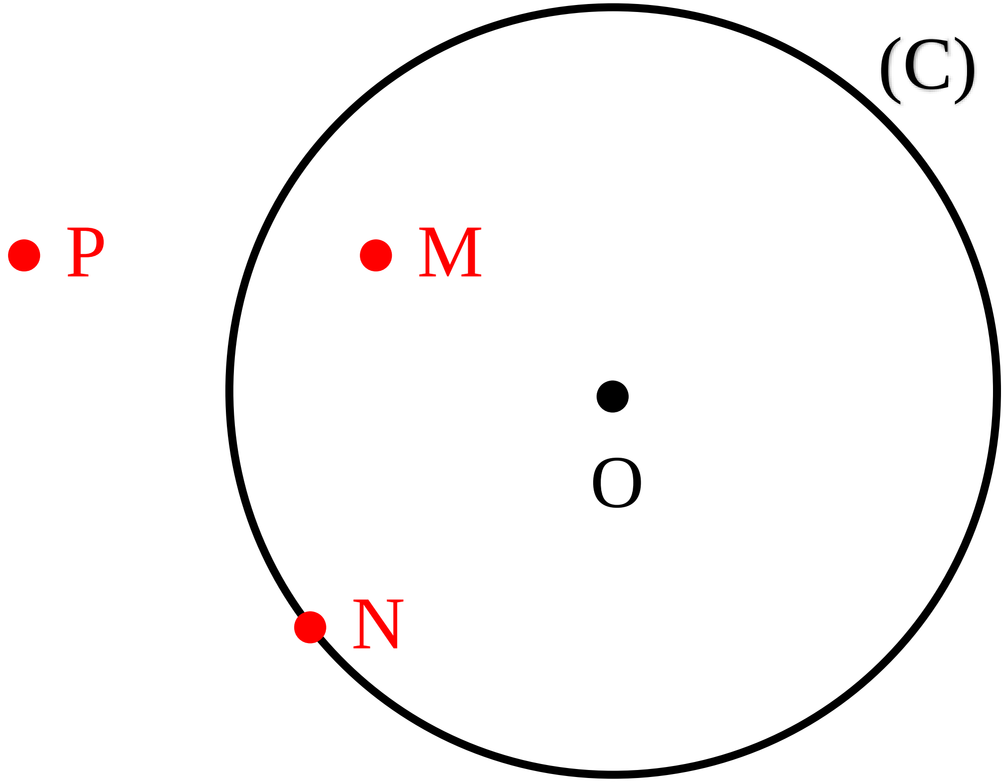
Lines and circle

Point and circle

Consider the circle C(O;R)



M is at the interior of (C): $OM < R$

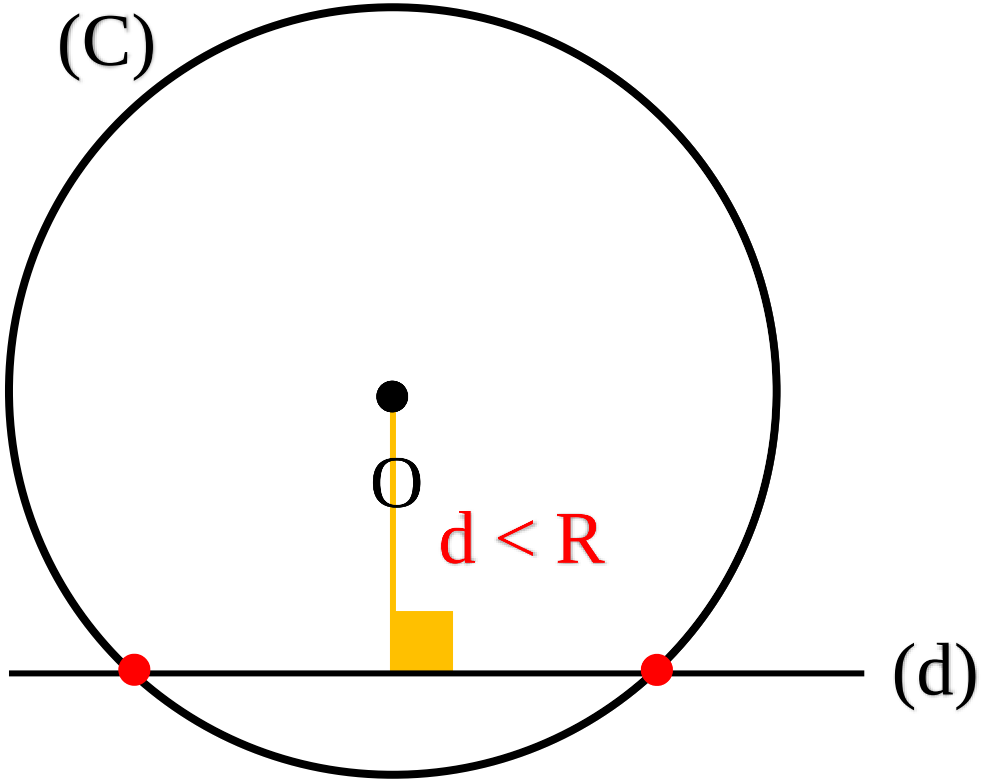
N is on (C): $ON = R$

P is at the exterior of (C): $OP > R$

Lines and circle

Consider the circle C(O;R)

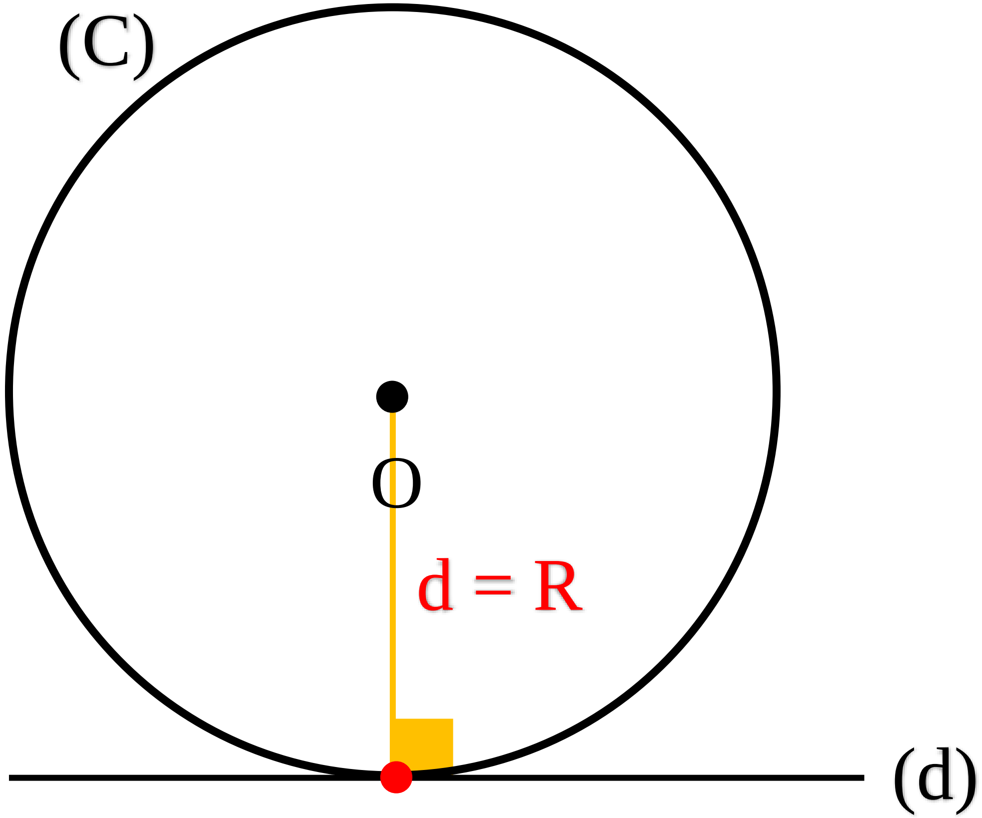
Secant



Lines and circle

Consider the circle C(O;R)

Tangent

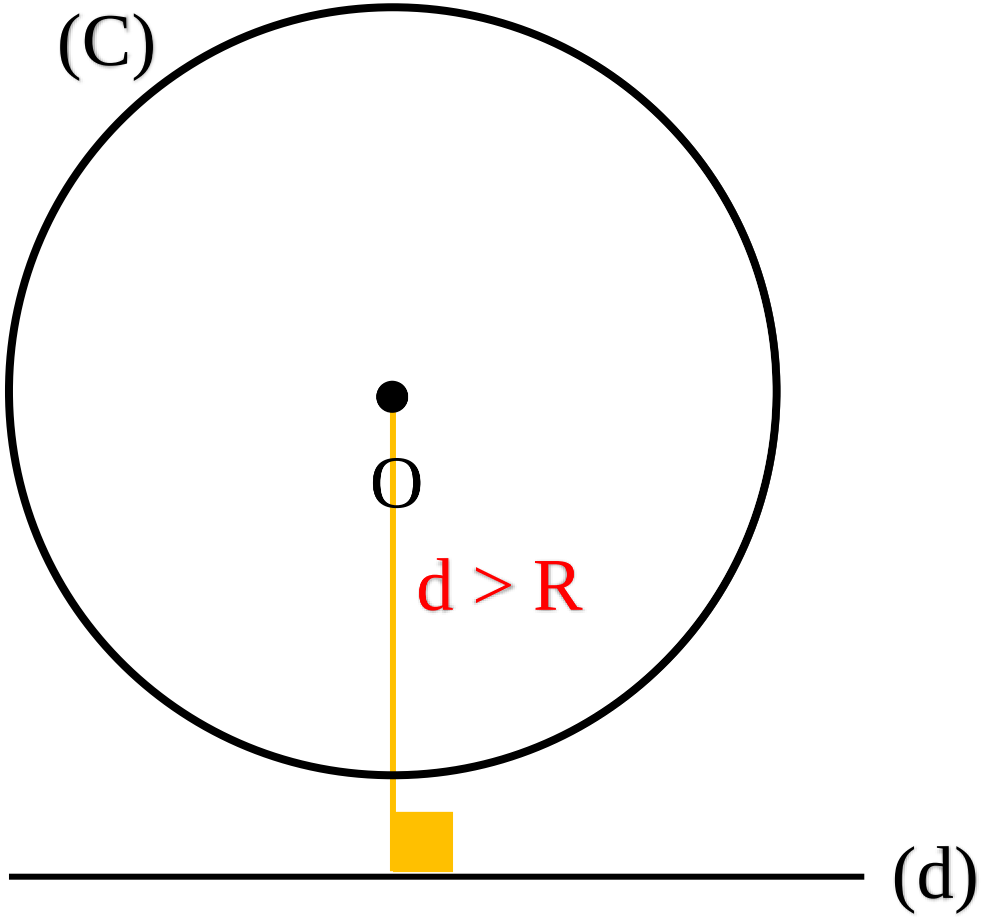


To prove that a line (d) is tangent:
Prove that (d) is perpendicular to the radius at the point of tangency

Lines and circle

Consider the circle C(O;R)

Exterior line



Lines and circle

Application # 1

ABC is a triangle of sides AB=5cm, AC=3cm and BC=4cm.

Show that (AC) is tangent to the circle (C) of center B and radius BC.

$AB^2 = 5^2 = 25$

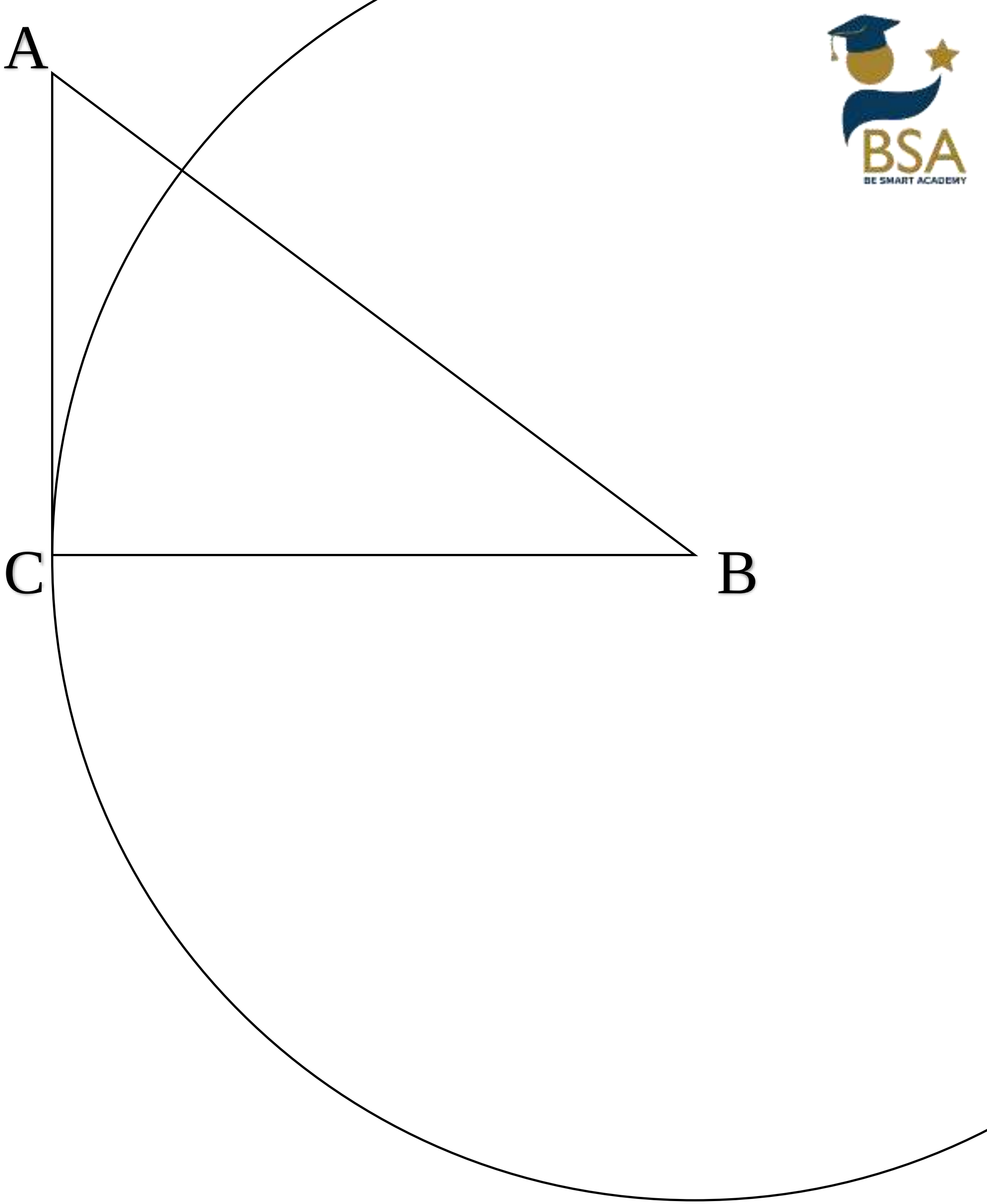
$AC^2 + BC^2 = 3^2 + 4^2 = 9 + 16$

$= 25 = AB^2$

According to the converse of Pythagoras theorem, ABC is a right triangle at C.

Then (AC) ⊥ (BC) at C

Hence (AC) is tangent to the circle (C).



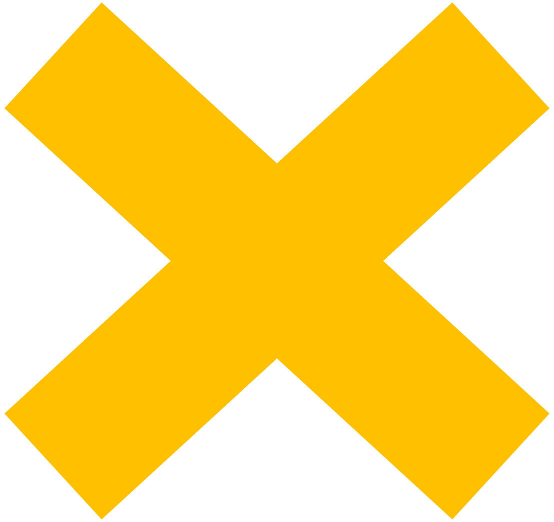
Tangents

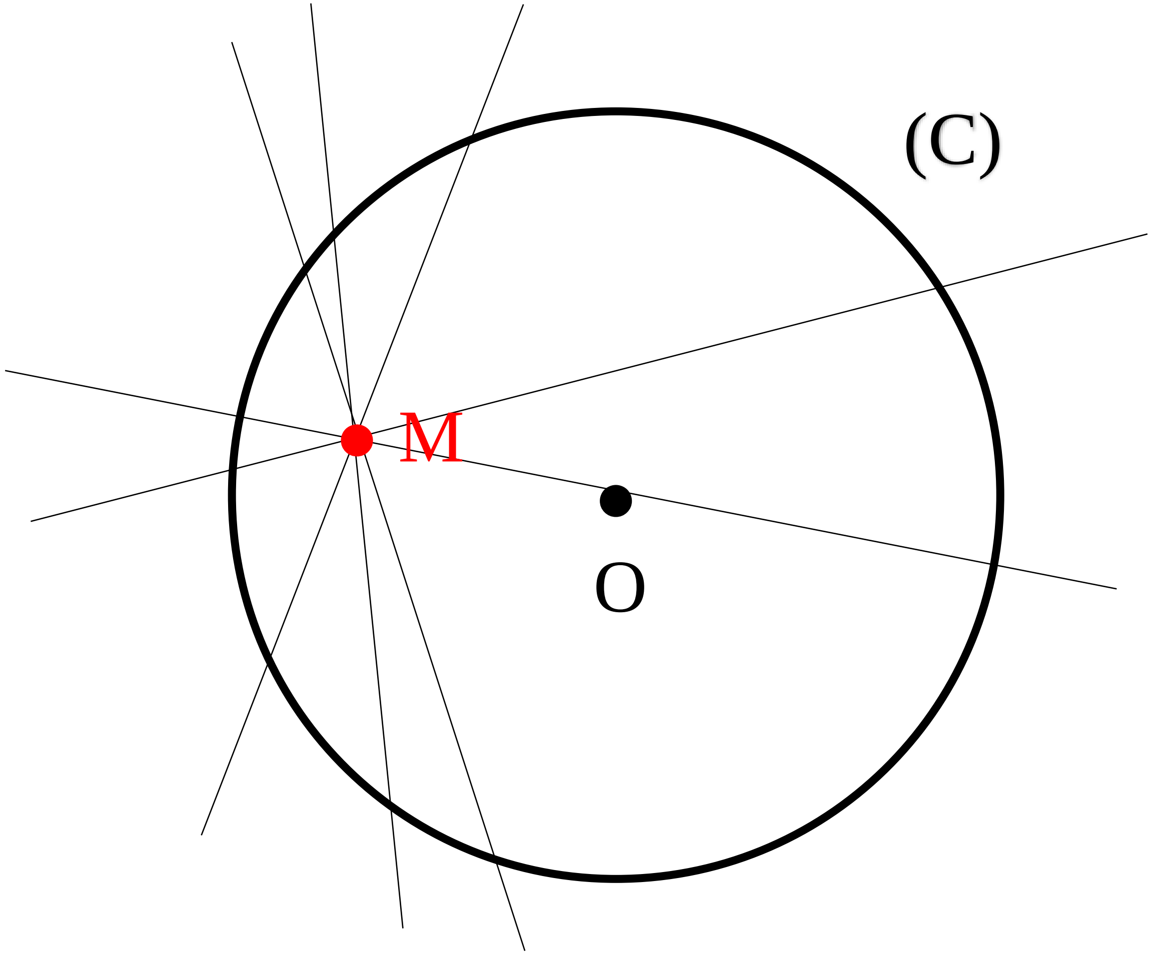
Consider the circle C(O;R)

How to draw a tangent from a given point to a given circle?

Case ❶

M is at the interior of (C)





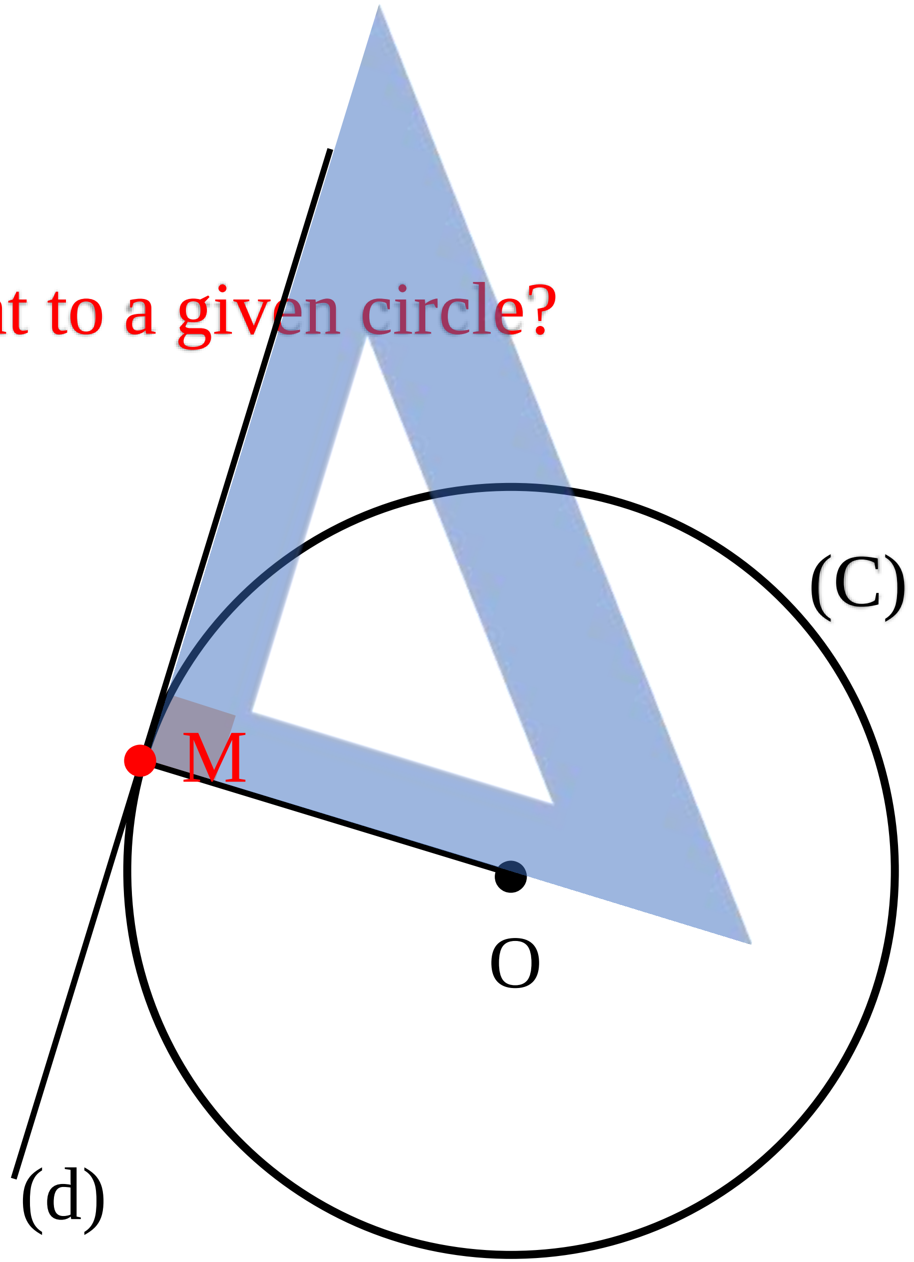
Tangents

Consider the circle C(O;R)

How to draw a tangent from a given point to a given circle?

Case ❷

M is on (C)



(d) is tangent to (C) at M

One tangent

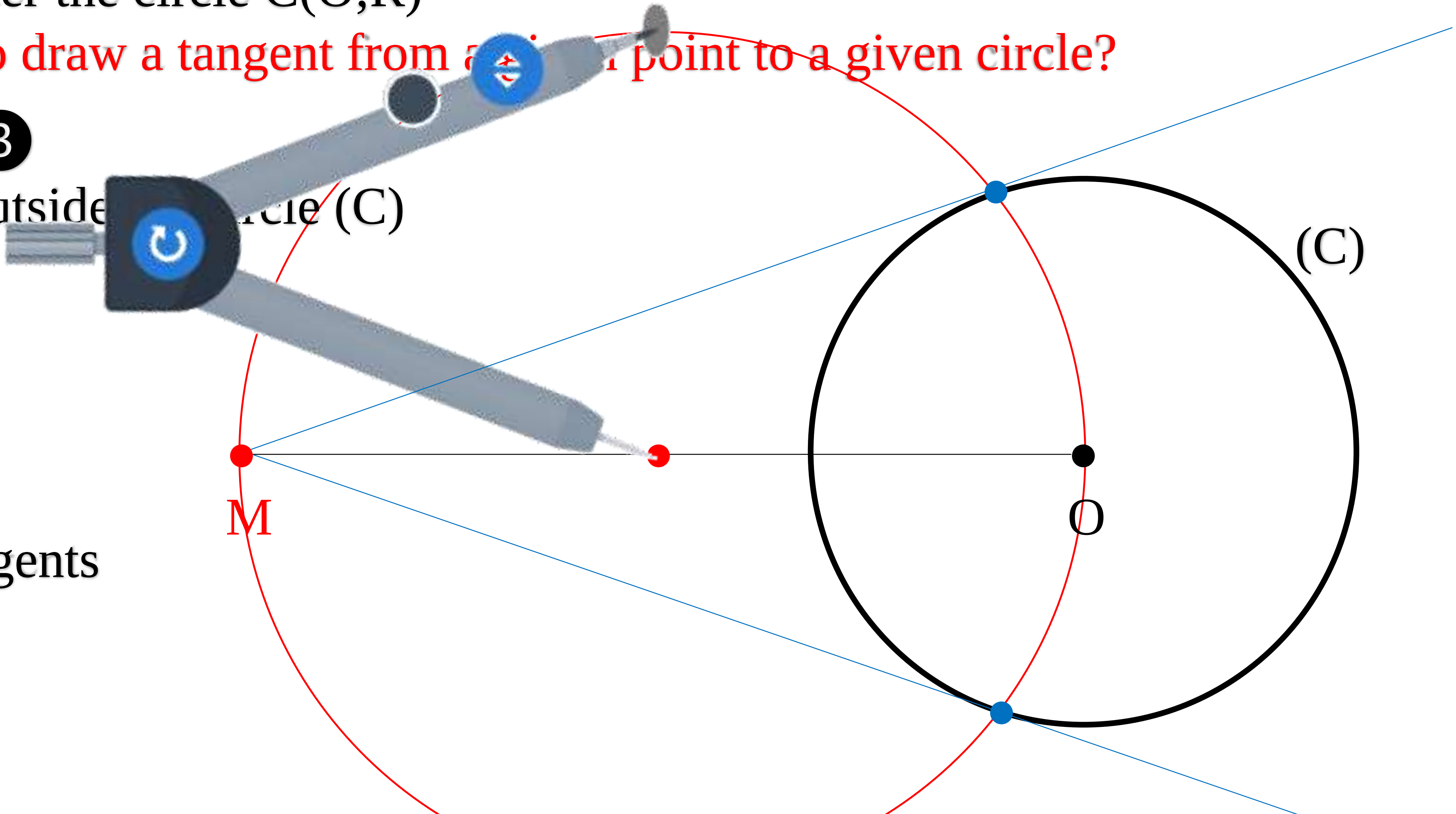
Tangents

Consider the circle C(O;R)

How to draw a tangent from a given point to a given circle?

Case ❸

M is outside the circle (C)



2 tangents

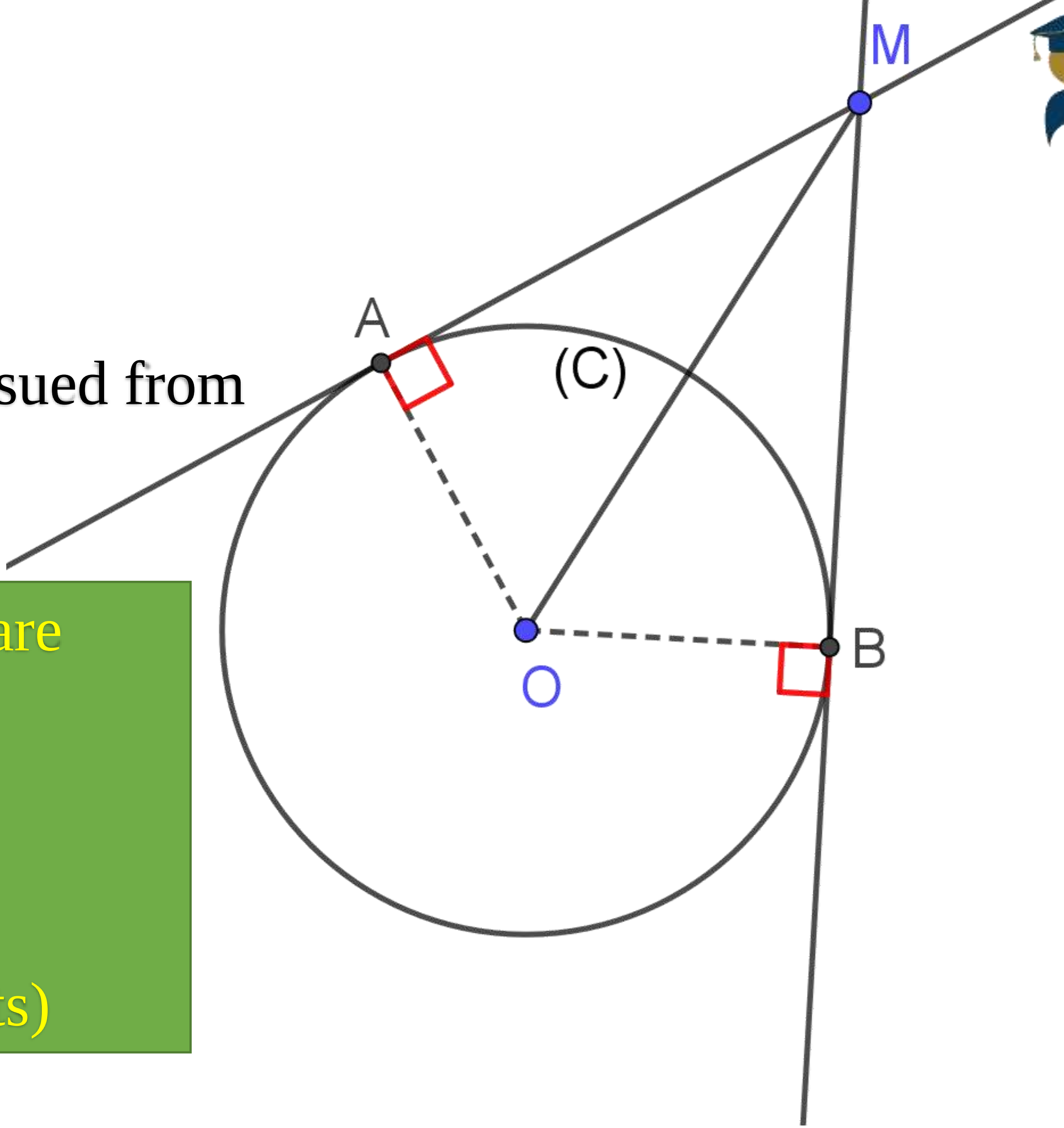
Tangents

Application # 2

Consider the circle C(O;R)

(MA) and (MB) are two tangents issued from M to (C) at A and B.

Show that MA=MB.



The two triangles MAO and MBO are congruent since:

AO = BO (radii)

[MO] is the common hypotenuse

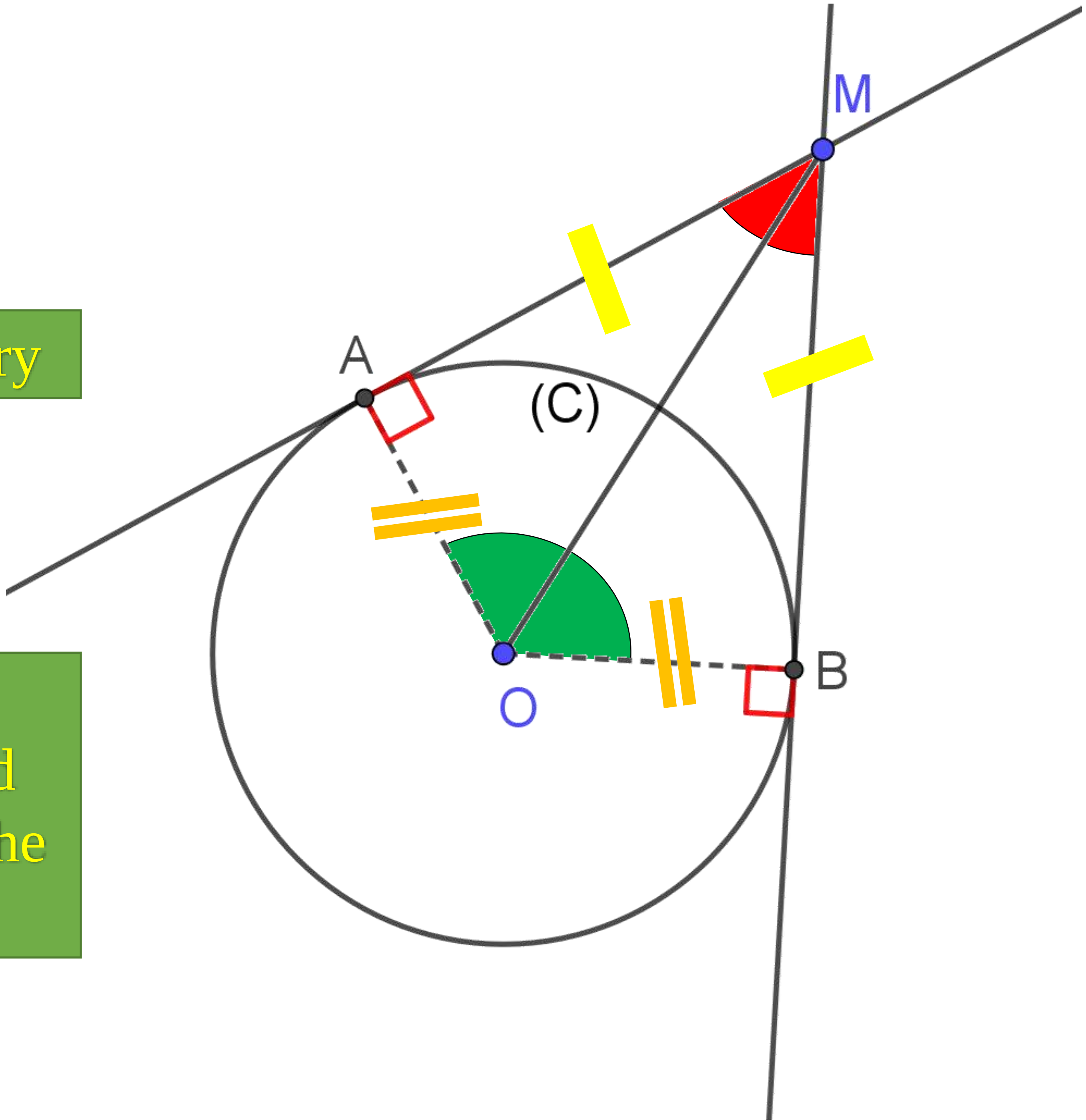
Then,

MA = MB (by homologous elements)

Tangents

(MO) is an axis of symmetry

In general,
MA=MB (2 tangents issued from the same point M to the same circle (C))

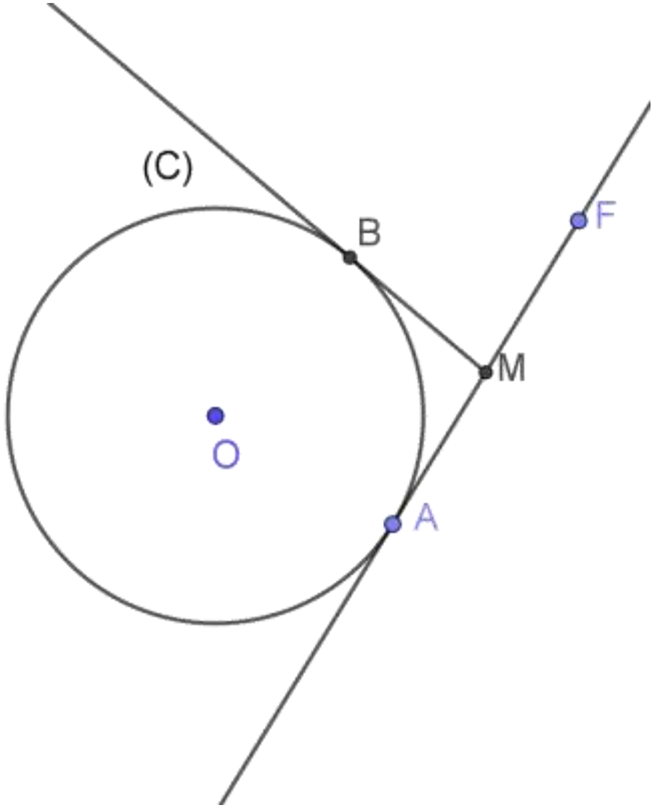


Lines and circle

Application # 3

In the following figure M is the midpoint of [FA] and (MA) and (MB) are the tangents issued from M to the circle (C).

Show that ABF is a right triangle.



In the triangle ABF, [BM] is a median.

MA=MB (2 tangents issued from the same point M to the same circle (C)).

MA=MF (M is the midpoint to [FA])

Then MB=MA=MF

Hence $BM = \frac{AF}{2}$

So the triangle ABF is a right triangle at B.